On Linear Variational Surface Deformation Methods

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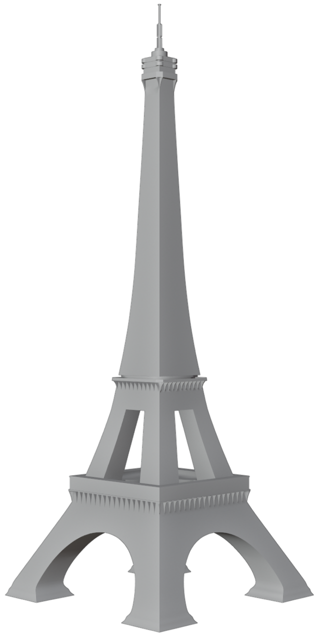
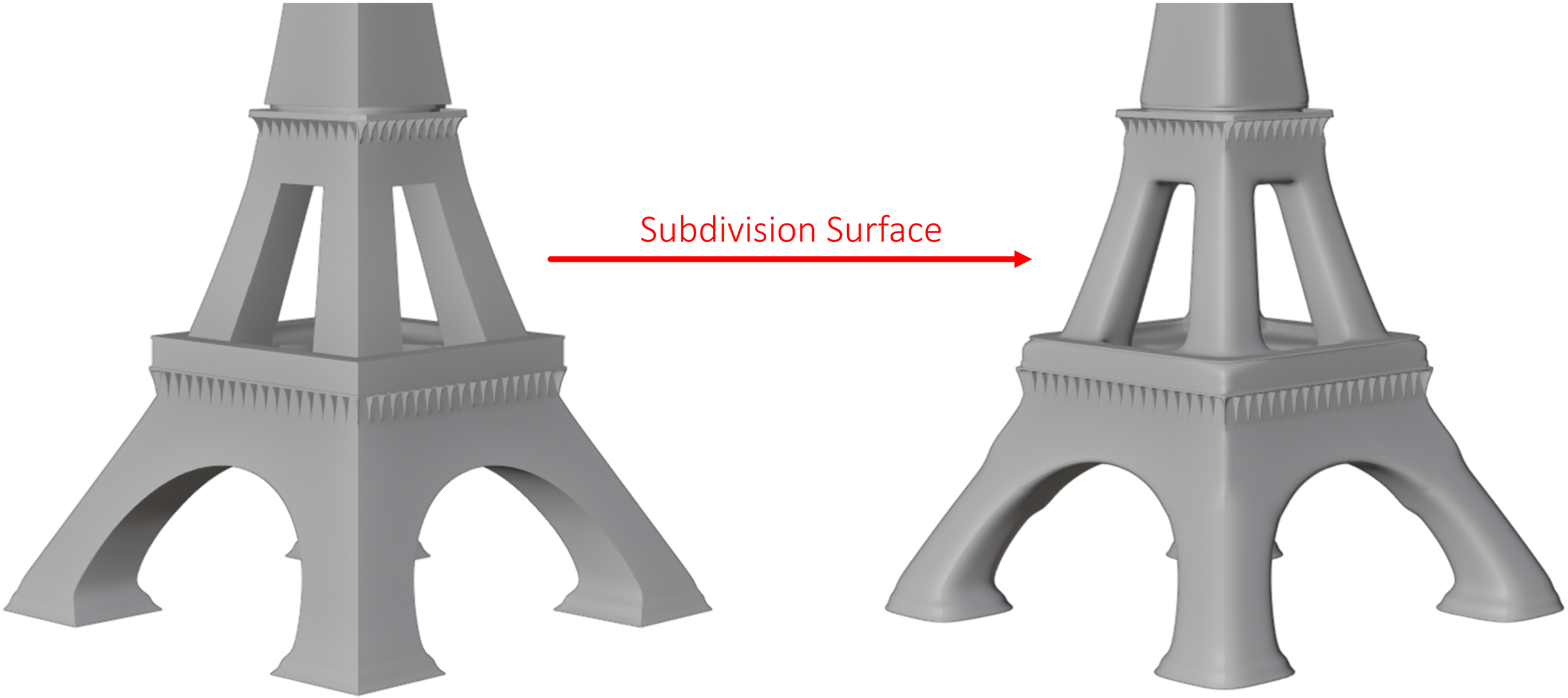


Not smooth
☹


Subdivision Surface

Multiresolution Editing

Continuous Formulation

$S \subset \mathbb{R}^3$ a two-manifold ($\sim$ surface in 3D or ‘thin shell’)

Parametrized by $p : \ \Omega \rightarrow S$

Deformation $p' = p + d, p' \in S'$

(S' is the deformed manifold)

Continuous Formulation

Elastic energy measuring deformation

$$E_{shell}(d) = \int_{\Omega} k_s(\| d_u \|^2 + \| d_v \|^2) + k_b(\| d_{uu} \|^2 + 2 \| d_{uv} \|^2 + \| d_{vv} \|^2) du dv$$

$$(u, v) \in \Omega$$

k_s = stiffness parameter for stretching

k_b = stiffness parameter for bending

Continuous Formulation

$(*)$

$$E_{shell}(d) = \int_{\Omega} k_s(\| d_u \|^2 + \| d_v \|^2) + k_b(\| d_{uu} \|^2 + 2 \| d_{uv} \|^2 + \| d_{vv} \|^2)dudv$$

Minimizing $(*)$ is equivalent to

$$-k_s \Delta d + k_b \Delta^2 d = 0$$

(Euler-Lagrange PDE)

Discretization

$$-k_s \Delta d + k_b \Delta^2 d = 0$$

(Euler-Lagrange PDE)

Finite differences method

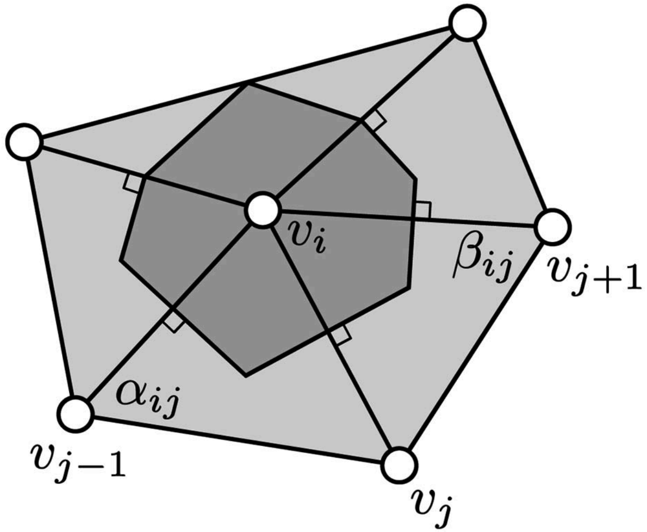
$$\Delta f(v_i) = w_i \sum_{v_i \in \mathcal{N}_1(v_i)} w_{ij}(f(v_j) - f(v_i))$$

Discretization

$$\Delta f(v_i) = w_i \sum_{v_i \in \mathcal{N}_1(v_i)} w_{ij}(\, f(v_j) - f(v_i)\,)$$

$$w_i = \frac{1}{A_i}$$

$$w_{ij} = \frac{1}{2}(\, \cot(\alpha_{ij}) + \cot(\beta_{ij})\,)$$



Numerical Solution

$$\begin{pmatrix} \Delta f(v_1) \\ \Delta f(v_2) \\ . \\ . \\ \Delta f(v_n) \end{pmatrix} = \underbrace{M^{-1} L_s}_{L} \begin{pmatrix} f(v_1) \\ f(v_2) \\ . \\ . \\ f(v_n) \end{pmatrix}$$

$$L_s = \begin{cases} \sum_{v_k \in \mathcal{N}_1(v_i)} w_{ik}, & i = j, \\ w_{ij}, & v_k \in \mathcal{N}_1(v_i), \\ 0, & otherwise \end{cases}$$

Numerical Solution

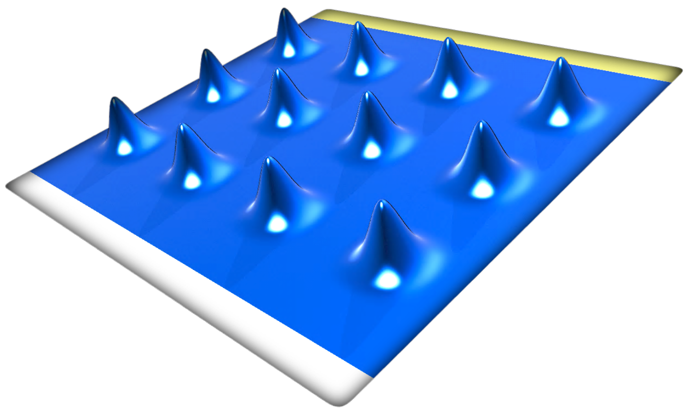
$$\begin{pmatrix} \Delta f(v_1) \\ \Delta f(v_2) \\ . \\ . \\ \Delta f(v_n) \end{pmatrix} = \underbrace{M^{-1} L_s}_{L} \begin{pmatrix} f(v_1) \\ f(v_2) \\ . \\ . \\ f(v_n) \end{pmatrix}$$

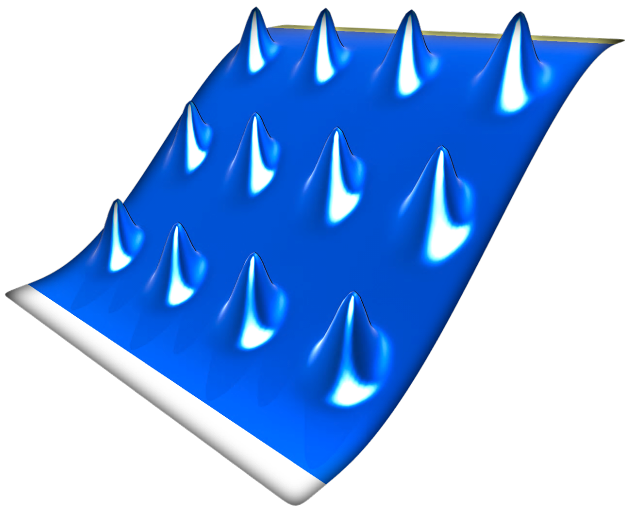
$$\boxed{\left(-k_s \boldsymbol{L} + k_b \boldsymbol{L}^{\mathbf{2}}\right) d = 0}$$

Limitations

$$\left(-k_s\boldsymbol{L} + k_b\boldsymbol{L^2}\right)d = 0$$

Problems occur when performing local rotations

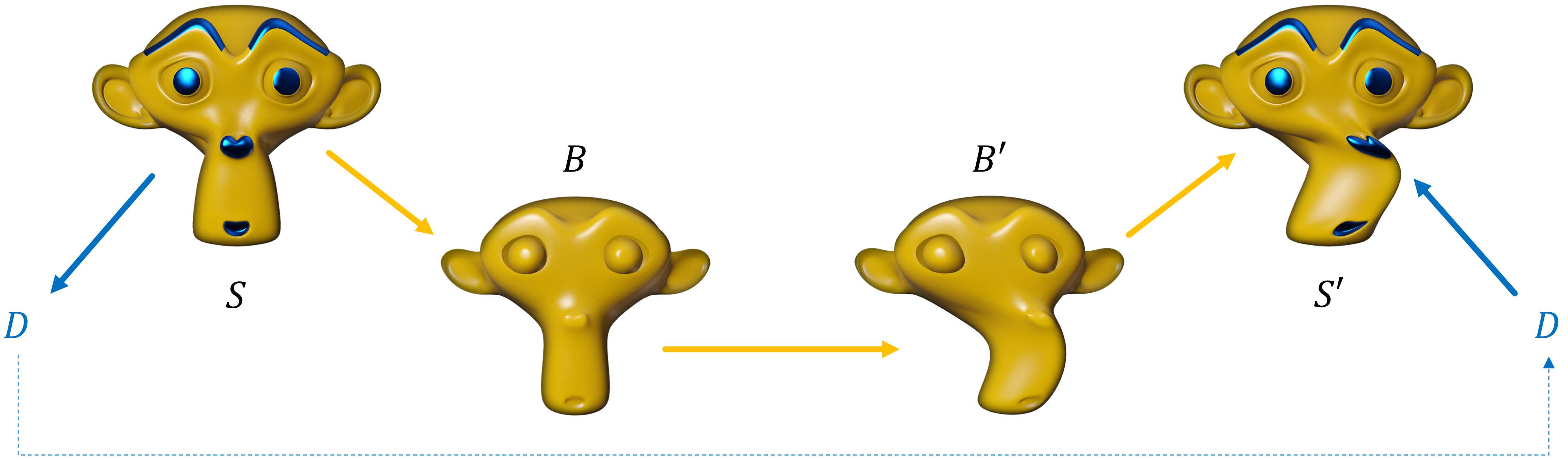




Multiresolution Hierarchies

Decompose S in low and high frequencies

$$S = B \oplus D$$

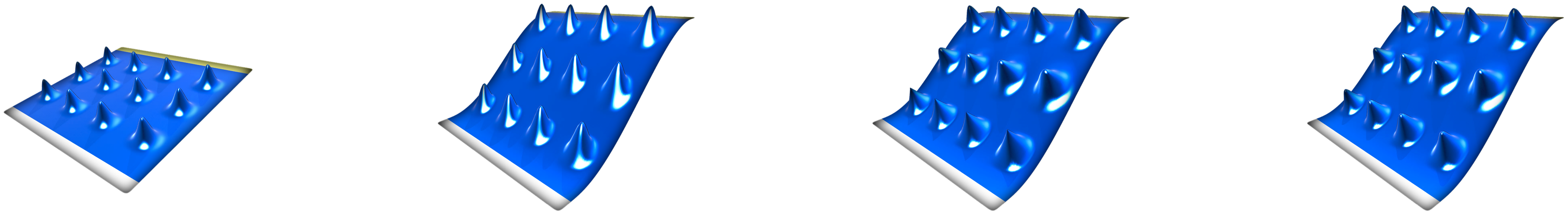


Multiresolution Hierarchies

Straightforward approach : $p_i = b_i + h_i$

Normal displacement method : $p_i = b_i + h_i \cdot n(b_i)$

Volume preservation



Thank you